

Stiffness of Symmetrical Bodies:-

$$\text{Stiffness} = \frac{\text{Load}}{\text{deformation}}$$

1) Solid Cylinder:

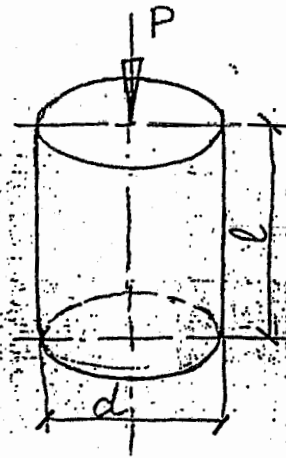
$$E = \frac{\text{stress}}{\text{strain}} = \frac{\sigma}{\epsilon}$$

$$\sigma = \frac{P}{\left(\frac{\pi d^2}{4}\right)} = \frac{P}{A}$$

$$\epsilon = \Delta l / l$$

$$\therefore E = \frac{P}{A} \cdot \frac{l}{\Delta l}$$

$$\text{or } C = \frac{P}{\Delta l} = \frac{EA}{l} = \frac{E}{l} \left[\frac{\pi d^2}{4} \right]$$



2) Hollow Cylinder:

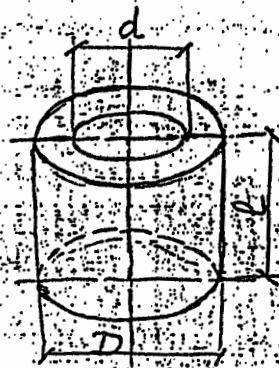
Outer diameter = D

Inner diameter = d

$$\text{and } A = \frac{\pi}{4} [D^2 - d^2]$$

Hence

$$C = \frac{EA}{l} = \frac{E}{l} \left[\frac{\pi}{4} (D^2 - d^2) \right]$$

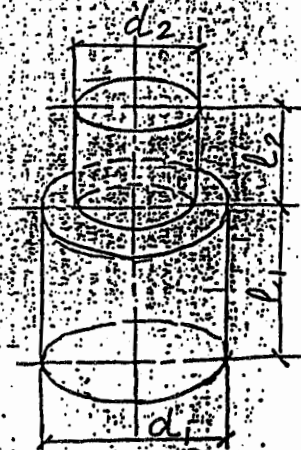


3) Two Cylinders in Series

$$C_1 = \frac{E_1 A_1}{l_1} = \frac{E_1}{l_1} \left[\frac{\pi}{4} d_1^2 \right]$$

$$C_2 = \frac{E_2 A_2}{l_2} = \frac{E_2}{l_2} \left[\frac{\pi}{4} d_2^2 \right]$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$



4) Frustum of Right Circular Cone:

Frustum diam^s are d and D

Frustum height = l

Equivalent Cyl diam = $\frac{d+D}{2}$

$$C = \frac{E}{l} \left[\frac{\pi (d+D)^2}{4} \right]$$

5) Frustum with axial hole:

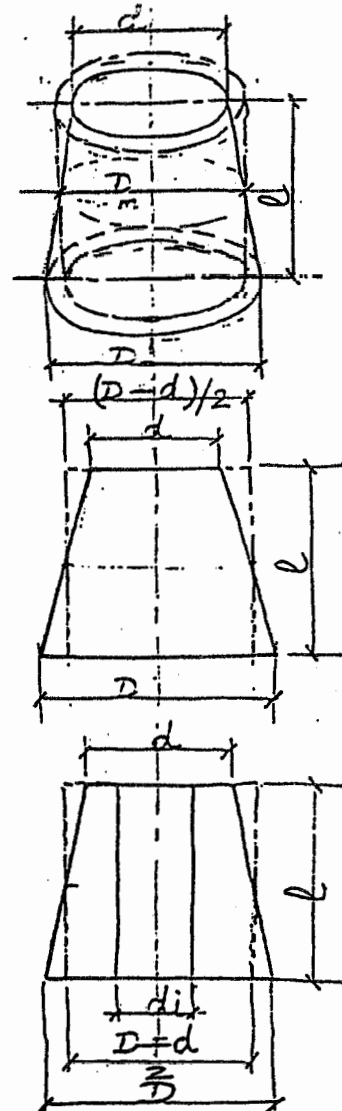
Central hole diam = d_i

$$C_i = \frac{E}{l} \left(\frac{\pi}{4} d_i^2 \right)$$

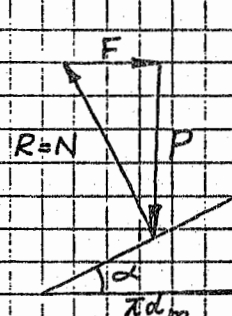
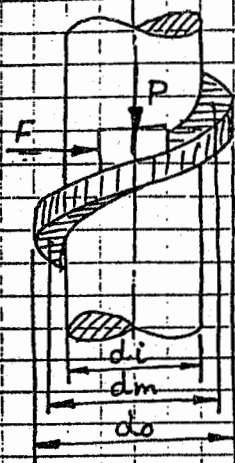
$$C_o = \frac{E}{l} \left[\frac{\pi}{4} \left(\frac{d+D}{2} \right)^2 \right]$$

$$C = C_o - C_i$$

$$= \frac{\pi E}{4 l} \left[\left(\frac{d+D}{2} \right)^2 - d_i^2 \right]$$

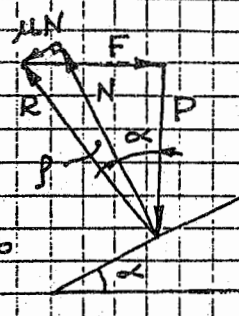


Forces on Axially Loaded Screw:



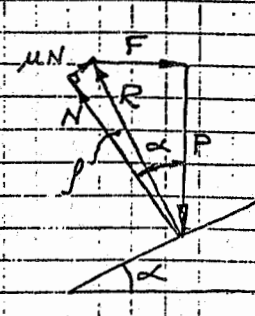
Frictionless

$$F = P \tan \alpha$$



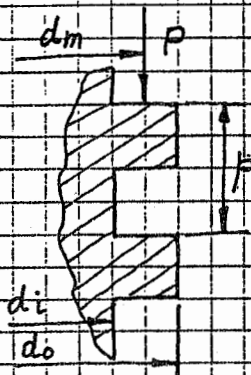
Lifting

$$F = P \tan(\alpha + \phi)$$

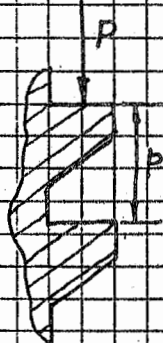


Lowering

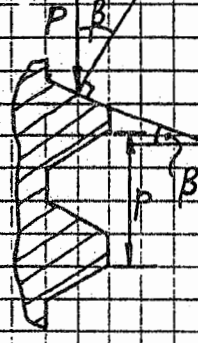
$$F = P \tan(\alpha - \phi)$$



Square



Buttress



Vee-thread.

Thread	β
Square	0
Buttress	0
Metric	30°
Whitworth	27½°
Trapezoidal	15°
Round	15°

$$\tan \phi = \mu$$

$$\tan \alpha = p / \pi d_m$$

For Square and Buttress Threads: (Lifting)

$$T_f - \text{Thread Friction Torque} = F \frac{d_m}{2} = P \frac{d_m}{2} \tan(\alpha + \phi)$$

$$= P \frac{d_m}{2} \left[\frac{\tan \alpha + \mu}{1 - \mu \tan \alpha} \right]$$

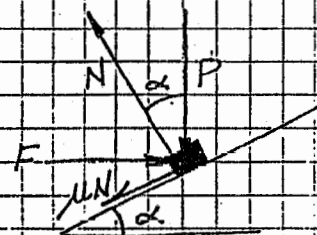
For Vee-Threads:

$$T_f = P \frac{d_m}{2} \left[\frac{\tan \alpha + (\mu / \cos \beta)}{1 - (\mu / \cos \beta) \tan \alpha} \right] = P \frac{d_m}{2} \left[\frac{\tan \alpha + \mu'}{1 - \mu' \tan \alpha} \right], \mu' = \frac{\mu}{\cos \beta}$$

$$F = N \sin \alpha + \mu N \cos \alpha \quad (\Sigma x = 0)$$

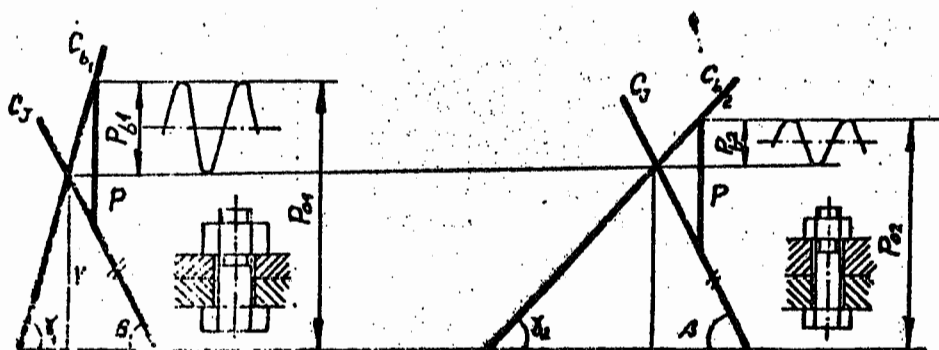
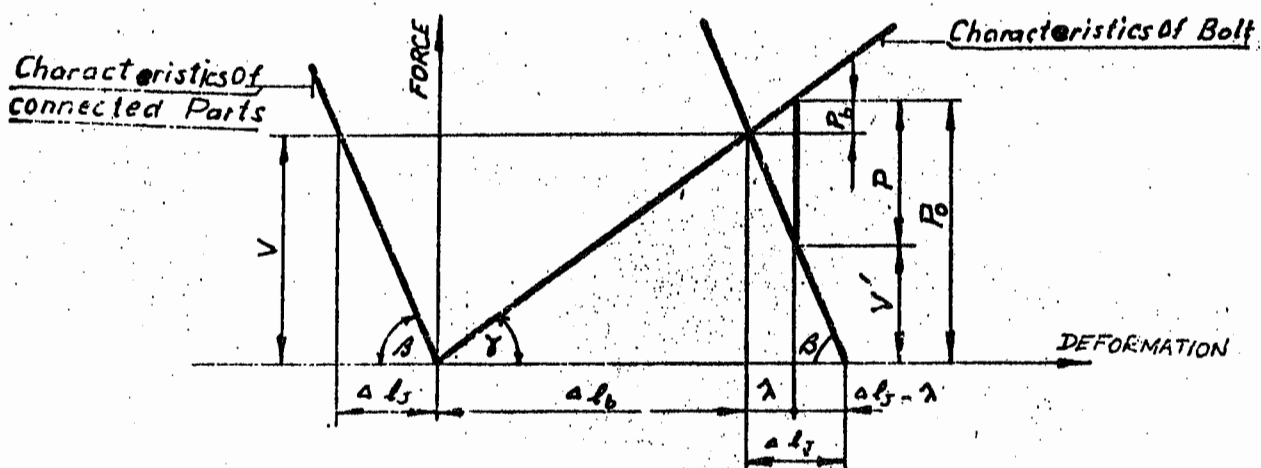
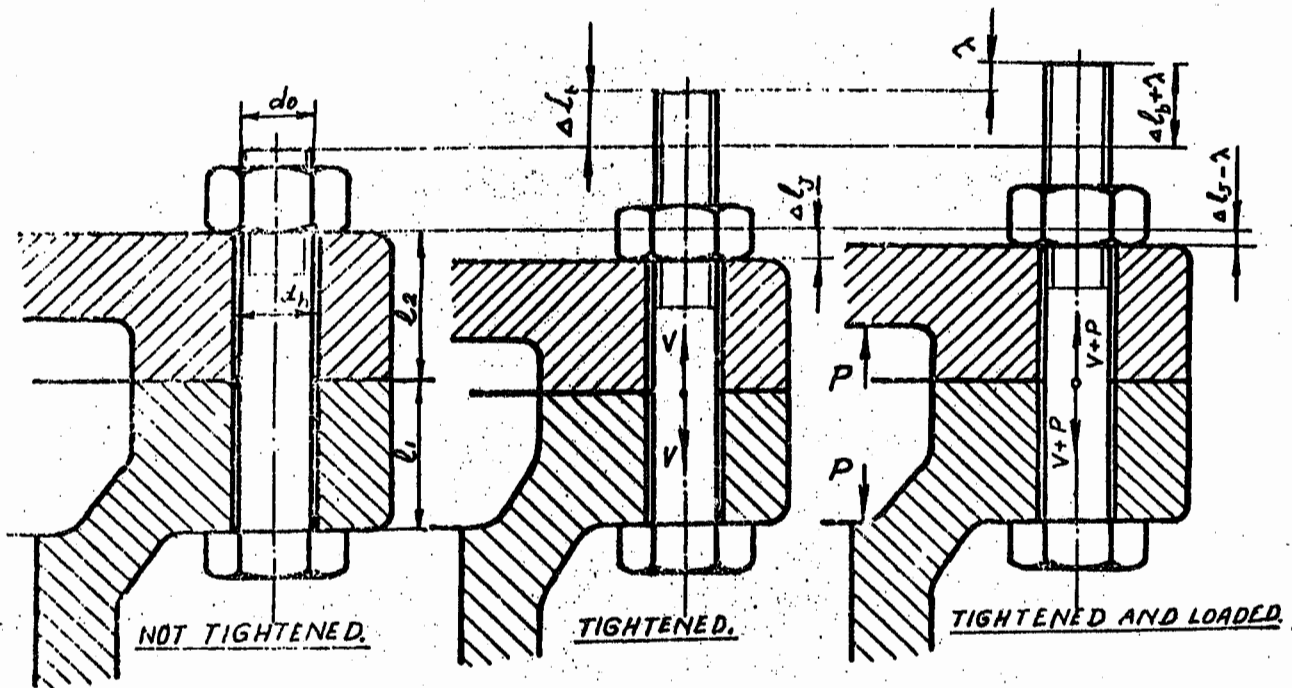
$$P = N \cos \alpha - \mu N \sin \alpha \quad (\Sigma y = 0)$$

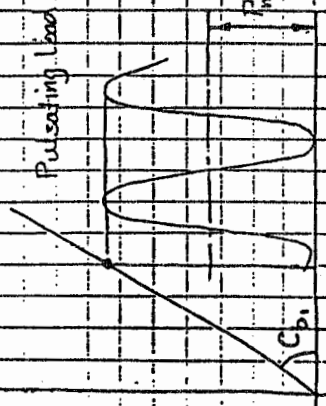
$$\frac{F}{P} = \frac{\sin \alpha + \mu \cos \alpha}{\cos \alpha - \mu \sin \alpha} = \frac{\tan \alpha + \mu}{1 - \mu \tan \alpha}$$



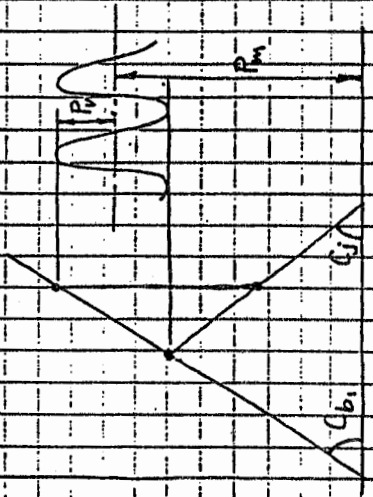
SQ. Thread

$$\text{Thread efficiency} = \eta = \frac{\tan \alpha}{\tan(\alpha + \phi)}$$

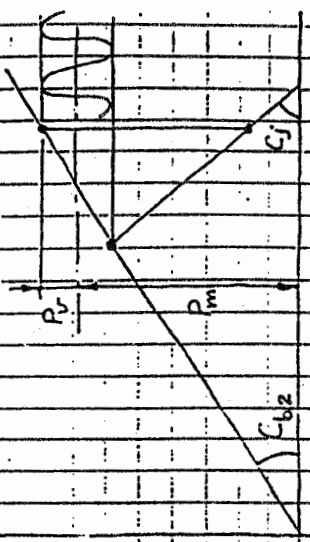




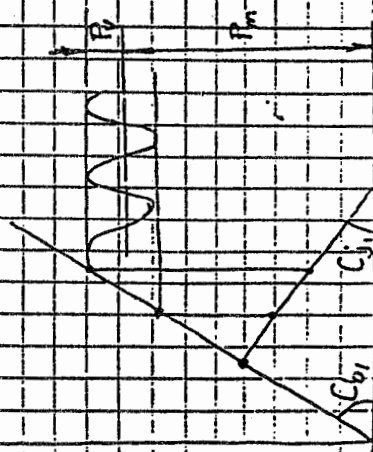
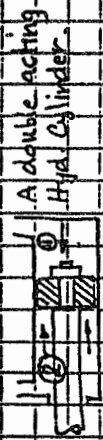
Without Preload



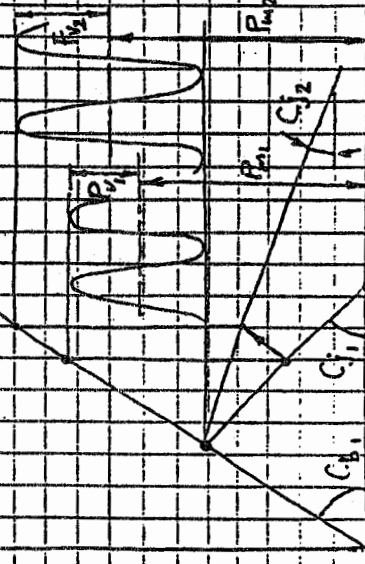
With Preload



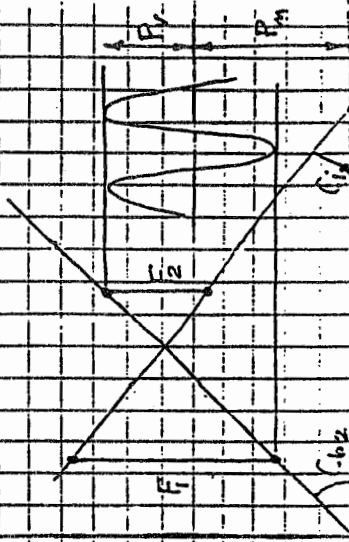
With Preload & Elastic bolt



Variable External load



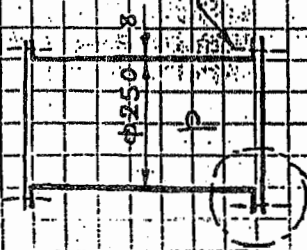
Softer joint, Some external Puls. load



Alternating External load

Bolted Joint under static and dynamic loads.

Ex:



p = Internal pressure = 2 M.Pa

NOTE:

$$\begin{aligned} 11 \text{ MPa} &= 0.6 \text{ N/mm}^2 \\ &= 0.1 \text{ N/mm}^2 \\ &= 0.1 \text{ kg/cm}^2 \\ &= 10 \text{ kg/cm}^2 \end{aligned}$$

Case 1: Const p

Case 2: $p_{\max} = 2 \text{ MPa}$, $p_{\min} = 0$

Case 3: $p_{\max} = 2 \text{ MPa}$, $p_{\min} = 0.93 \text{ MPa}$

kN Load

15



$P_{\text{initial}} = 10 \text{ kN}$

$12 \text{ M } 16 \times 2$
Grad 10.9

$P_{\text{ext}} = 8.6 \text{ kN}$

7.26

6.8

$P_{\text{res}} = 4.5 \text{ kN}$

4 kN

$\theta_6 = \tan^{-1} C_6$

$\tan C_3$

20

10

0

10

20

25

$\text{mm} \times 10^{-6} \text{ mm}$

$\text{mm} \times 10^{-6} \text{ mm}$

Deformation